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MATERIEL MANAGEMENT STUDIES

INVENTORY POLICIES FOR A MULTI-ECHELON SUPPLY SYSTEM WITH FIXED ORDER COSTS AT LOWER ECHELONS

PRC

PLANNING RESEARCH CORPORATION
LOS ANGELES, CALIFORNIA

INVENTORY POLICIES FOR A MULTI-ECHELON SUPPLY SYSTEM WITH FIXED ORDER COSTS AT LOWER ECHELONS

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By

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ABSTRACT

This report concerns the problem of approximating optimum (least-cost) inventory policies for a multi-echelon supply system with fixed order costs at lower echelons. First, the technique for computing the approximate policies is explained in nonmathematical terms. The sensitivity of these policies, and their resulting costs, to changes in parameter values is then investigated. Finally, the effect of implementing these approximate policies in a multi-echelon supply system is studied in an effort to determine the cost increase over the theoretically optimum cost. The resulting cost increase is less than 3 percent for a typical inventory.

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I. INTRODUCTION

This report concerns the problem of approximating least-cost inventory policies for a multi-echelon supply system with fixed order costs at lower echelons. Approximations are used because finding truly least-cost solutions requires the consideration of two or more state variables, and, while such solutions are theoretically possible, the computation involved is too lengthy to be economically feasible.

The problem of determining least-cost inventory policies for a multi-echelon supply system was first considered by Clark, who formulated the problem in a dynamic programming structure (Reference 1). This formulation, however, assumed fixed order costs to be zero at lower echelons. Later Clark and Scarf considered the mathematical aspects of the formulation and established conditions under which the policies determined were truly least-cost (Reference 2). This author later investigated the nature of solutions to the multi-echelon inventory problem through case studies (Reference 3).

Recently Clark modified the first formulation to accommodate fixed order costs at lower echelons and yield policies which approximate the least-cost policies. This report deals with several aspects of this new formulation. In Section II the technique for computing the policies is described in nonmathematical terms. In Section III the sensitivity of policies and total inventory costs to parameter values is investigated. In Section IV the practical usefulness of such policies is evaluated, and areas for further investigation are suggested in Section V.

II. COMPUTATIONAL TECHNIQUE

To explain the technique for computing inventory decision levels in a multi-echelon structure with fixed order costs at lower echelons, a simple example may be considered. Suppose there is a multi-echelon structure consisting of a single base that obtains all its resupply from a single depot. For the moment, assume the base incurs no fixed order cost when placing an order.

A. Without Fixed Order Cost at Base

Exhibit 1 shows a purely arbitrary cost curve[1] for the base if resupply is not obtained. The curve shows the expected future costs that

[1]Actually, this curve and others presented in the subsequent discussion are shown for illustrative purposes only. Since most stock balances are confined to an integral number of units, only the points on the curves corresponding to such integral units of stock balance have meaning.

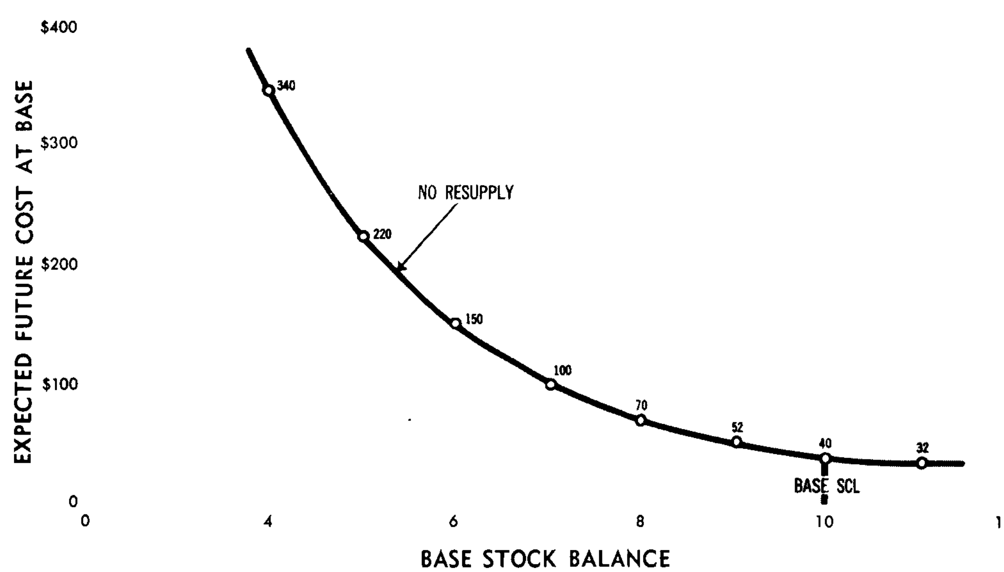

EXHIBIT 1. NO RESUPPLY CURVE FOR BASE (WITHOUT FIXED ORDER COST AT BASE)

will be incurred for each possible base stock balance. The first problem is to determine what the stock balance should be from an economic standpoint. If the items of inventory for which the curve is drawn cost $10 each, the object is to find the last additional unit of the item which, if purchased, will save $10 or more in future cost. Purchase of the fifth unit certainly saves $10 or more. In fact, purchase of the fifth unit saves $120, since the future cost is $340 with four units and $220 with five units. Also, each of the sixth through tenth units saves $10 or more when purchased. But purchase of the eleventh unit saves only $8, so it is economically unsound to spend $10 for that unit. The last additional unit purchased that will save $10 or more is the tenth, so, by definition, the stock control level for the base is 10.

The stock control level indicates that, if the stock falls below 10, enough units should be purchased to bring the stock back up to 10. Since an order must be placed if the stock falls below 10, it is easy to construct a cost curve for the base if resupply is obtained. Suppose the stock balance is 9, and resupply is obtained; the stock control level indicates that one unit of the item should be purchased for $10 to bring the stock balance up to 10. The cost after resupply associated with the stock balance of 9 is therefore $10 plus the future cost of $40 associated with the new stock balance of 10 created by the purchase. The total is $50. Similarly, the cost after ordering associated with a stock balance of 8 is $20 + $40 = $60; with a stock balance of 7 it is $70, and so forth. The curve (actually a straight line) representing cost after resupply for the base is therefore as indicated in Exhibit 2.

Suppose the base finds itself with 6 units on hand, and dutifully places an order on the depot for the 4 units needed to bring its balance up to 10. What if the depot does not have 4 units to fill the order? The base then must endure with 6 units, and incur a future cost of $150 instead of the $80 future cost applicable if the order had been filled. Because of the depot's inability to fill the order the base has incurred an extra $70 in future cost. The depot, therefore, should rightfully be charged with the $70. This $70 is a shortage cost implied to the depot

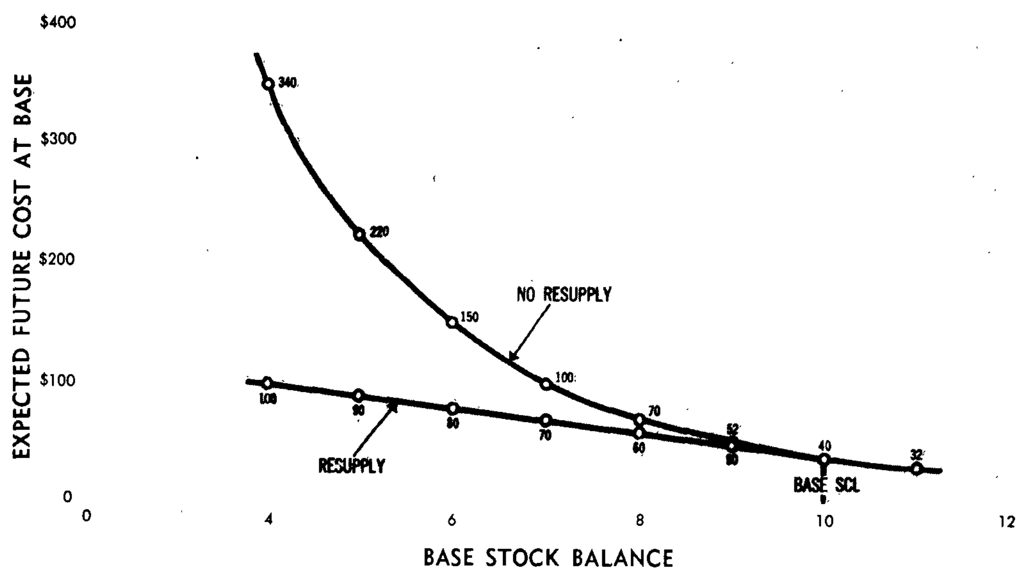

EXHIBIT 2. RESUPPLY AND NO RESUPPLY CURVES FOR BASE (WITHOUT FIXED ORDER COST AT BASE)

when a total of 6 units exists at and below the depot, irrespective of their distribution between the base and the depot.[1] This cost is, by definition, the implied shortage cost.

The depot is charged with an implied shortage cost any time the total number of units at the base and depot (irrespective of their distribution) is less than 10. This is because the base will be below its stock control level and must obtain resupply. Since the depot cannot provide resupply it must be charged with the extra cost incurred by the base.

[1] The independence of the implied shortage cost on the distribution of stocks between base and depot can be verified by noting the results when the base has 2 units and the depot has 4. The base places an order for 8 units but the depot can only partially fill the order, bringing the base balance up to 6. The implied shortage cost is still $70, since the base will incur $150 in future cost when it should incur only $80 at a stock balance of 6.

This extra cost, the implied shortage cost, is always the difference between the no resupply and resupply curves; that is, the shaded area in Exhibit 3.

When plotted by itself, the implied shortage cost appears as shown in Exhibit 4. It is taken in this form and added to the purely arbitrary depot future cost curve (with no resupply) as presented in Exhibit 5. The operatiคn for determining the depot stock control level is then similar to that for the base.

B. With Fixed Order Cost at Base

The computational technique is somewhat more involved when a fixed order cost is charged at the base. Suppose the base must incur a fixed order cost of $40 every time an order is placed, irrespective of the number of units ordered. Intuitively, the policy of ordering only one or two $10 units may be uneconomical if a $40 fixed order cost must be paid.

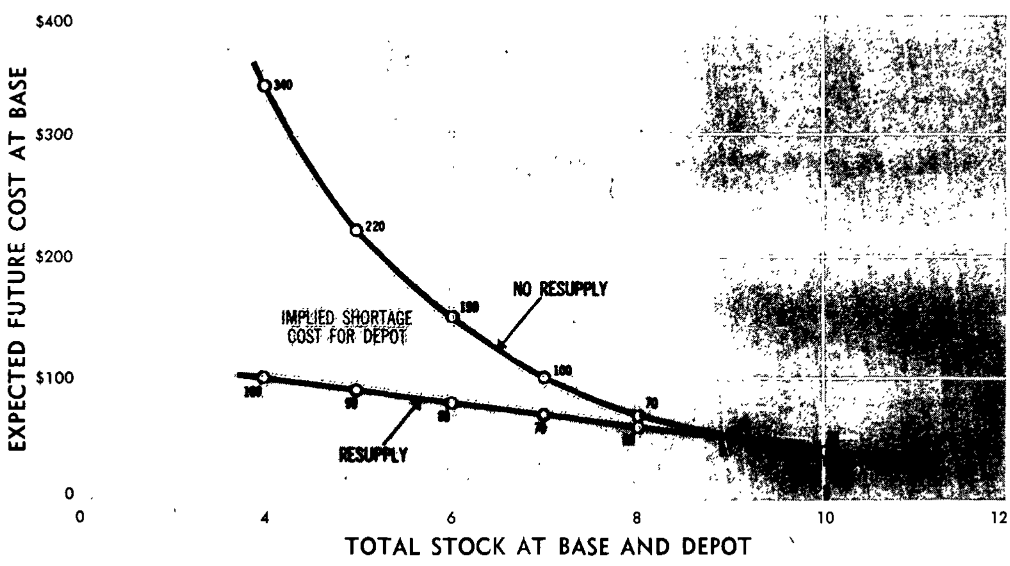

EXHIBIT 3. RESUPPLY AND NO RESUPPLY CURVES FOR BASE SHOWING IMPLIED SHORTAGE COST FOR DEPOT (WITHOUT FIXED ORDER COST AT BASE)

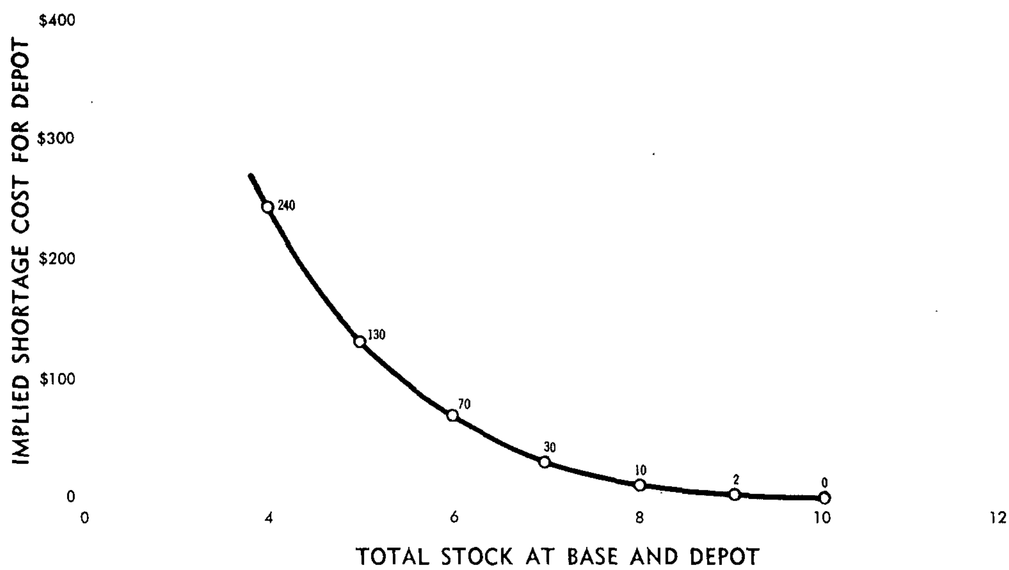

EXHIBIT 4. IMPLIED SHORTAGE COST FOR DEPOT (WITHOUT FIXED ORDER COST AT BASE)

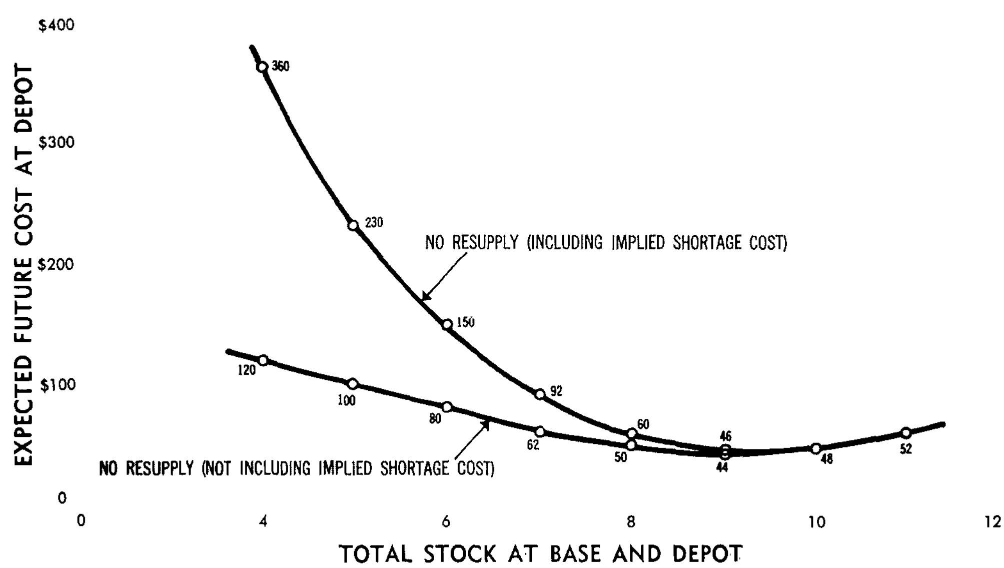

EXHIBIT 5. NO RESUPPLY CURVE FOR DEPOT INCLUDING AND NOT INCLUDING IMPLIED SHORTAGE COST (WITHOUT FIXED ORDER COST AT BASE)

The exact minimum economical size of the order may be found by considering the cost curve shown in Exhibit 1. The stock control level for the base is found in the same manner as before: by finding the last additional unit ordered that will save $10 or more in future cost. So the stock control level is still 10 in spite of the introduction of fixed reorder cost into the problem.

The line showing future cost after resupply must now be drawn. Since a fixed order cost of $40 is charged each time an order is placed, the line lies exactly $40 above its counterpart in Exhibit 2. Placement of the line is shown in Exhibit 6. Note in Exhibit 6 that for some values of the balance, namely 7 through 9, the no resupply line lies above the resupply curve. This indicates that it is uneconomical to obtain resupply when the base stock balance is 7 or more. The stock balance of 6 is, by definition, the <u>reorder point</u>. The stock control level minus the

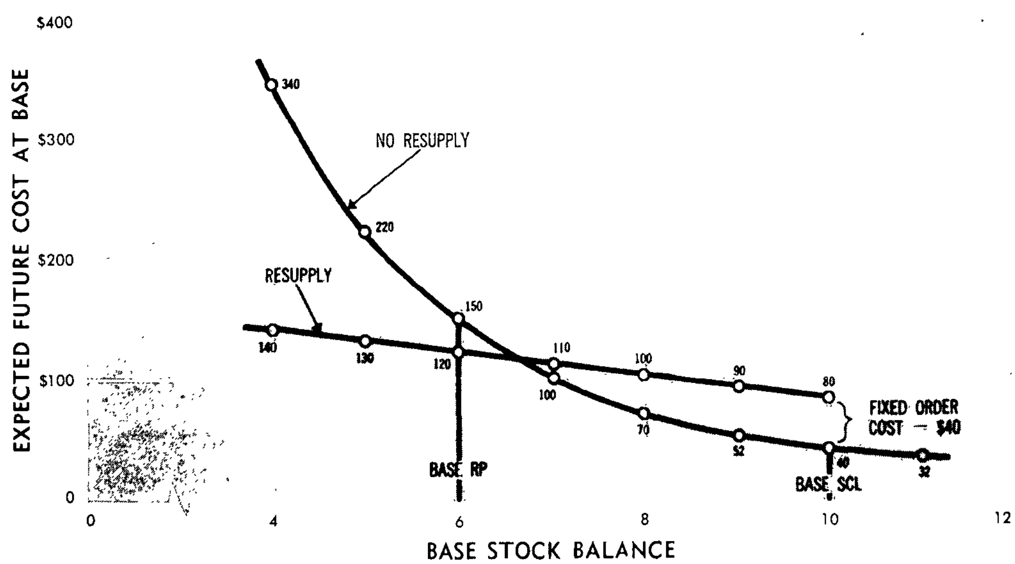

EXHIBIT 6. RESUPPLY AND NO RESUPPLY CURVES FOR BASE (WITH FIXED ORDER COST AT BASE)

reorder point signals the minimum economic order quantity, in this case 4. The rule for ordering is to order up to 10 when the stock falls to 6 or less.

The implied shortage cost must now be computed. Assume first that the total stock at the base and depot is at or below the base reorder point; that is, 6 or less. If the total base and depot stock were 5 units the base would place an order for the difference between its stock balance and 10, and the implied shortage cost would be, from Exhibit 6, the future cost of no resupply associated with a base stock balance of 5 <u>less</u> the future cost associated with 10 <u>less</u> the money saved by not purchasing the 5 items required to bring the base balance up to 10. The implied shortage associated with a total base and depot stock balance of 5 is then \$220 - \$40 - \$50 = \$130.[1] Note that this implied shortage cost is identical to that associated with the total base and depot stock of 5 when there is <u>no</u> fixed reorder cost as shown in Exhibit 4. In fact, whenever the total base and depot stock is at or below the base reorder point, the implied shortage cost will be identical to that obtained under the condition of no fixed order cost. This is because the base is sure to be at or below its reorder point and must place an order.

Assume now that the total base and depot stock is above the base reorder point. The implied shortage cost is now more difficult to compute, because it is no longer independent of the distribution of stock between the base and depot. Consider this example to show the dependency of the implied shortage on the stock distribution: suppose there are 8 units altogether at the base and depot. If the base has 7 or 8 of these units it does not place an order because it is above its reorder point of 6; therefore, no implied shortage cost is chargeable to the depot. However, if the base has 6 or less (leaving the depot with 2 or more) it must place an order for enough units to bring its balance up to 10. Its balance, however, will be brought up only to 8 and the depot, accordingly, must be charged with an implied shortage cost.

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[1] It is easily verified, by the same method as before, that this result is independent of the stock distribution between base and depot.

In fact, whenever the total number of units distributed between the base and the depot lies in the interval [(reorder point + 1) to (stock control level - 1)], there is some question as to whether or not to charge an implied shortage cost to the depot. The implied shortage cost should be charged if the base is at or below its reorder point and should not be charged if the base is above its reorder point.

Since the distribution of stock between the base and depot can be taken account of only by considering two state variables (namely, the base and depot stock balances), a method computationally infeasible, it is unknown whether or not the implied shortage cost should be charged to the depot.

One solution to this dilemma is to solve the problem in two different ways, one way assuming that the base is always at or below its reorder point and the other way assuming that the base is always above its reorder point. By assuming that the base is always at or below its reorder point, a maximum possible implied shortage cost is generated; and by assuming that the base is always above its reorder point, a minimum possible implied shortage cost is generated. These two costs are, by definition, the maximum implied shortage cost and the minimum implied shortage cost.

The maximum and minimum implied shortage costs for the problem at hand are computed by drawing an "effective" resupply line. This line is positioned just as if there were no fixed order cost at the base, and is used only in computing the implied shortage cost--it is not used in determining the reorder point. The maximum and minimum implied shortage costs are shown in Exhibits 7 and 8. When plotted by themselves they are as shown in Exhibit 9; note that they coincide with each other and with Exhibit 3 when the total stock at the base and depot is at or below the base reorder point.

The depot stock control level and reorder point are actually computed twice with independent computations: first with only the maximum implied shortage cost added to the depot future costs and then with only the minimum implied shortage cost added to the depot future costs. The

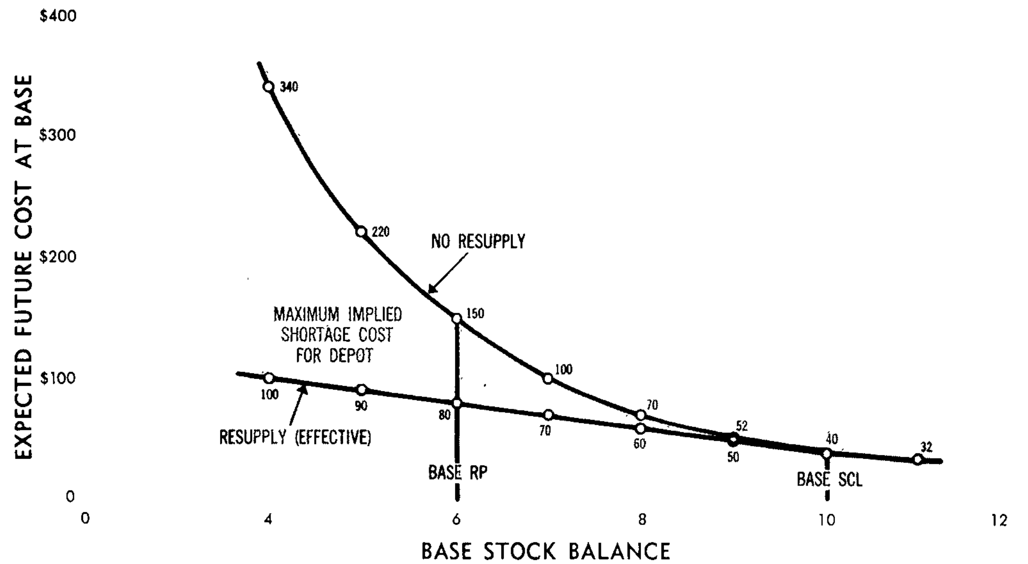

EXHIBIT 7. RESUPPLY (EFFECTIVE) AND NO RESUPPLY CURVES FOR BASE SHOWING MAXIMUM IMPLIED SHORTAGE COST FOR DEPOT

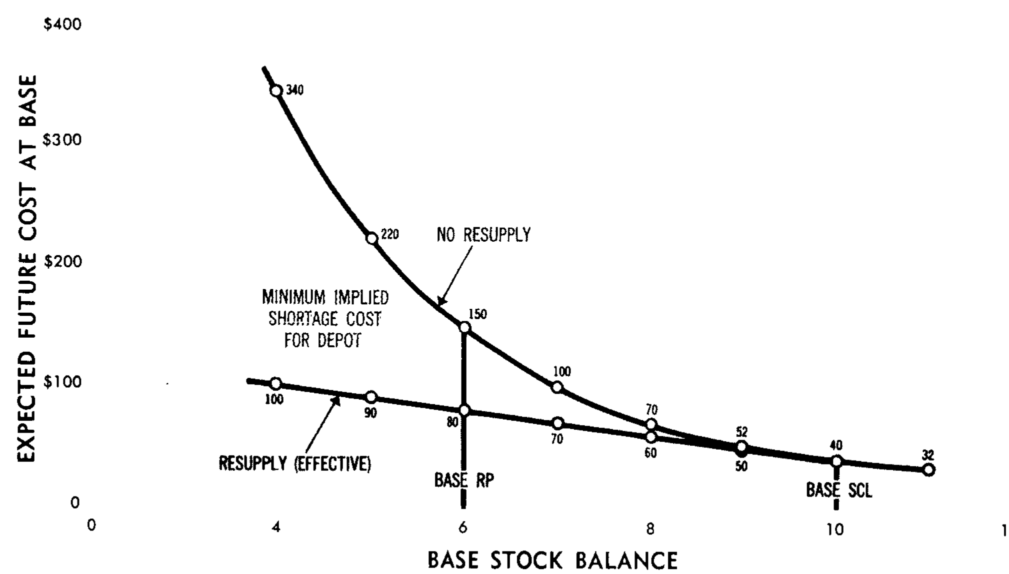

EXHIBIT 8. RESUPPLY (EFFECTIVE) AND NO RESUPPLY CURVES FOR BASE SHOWING MINIMUM IMPLIED SHORTAGE COST

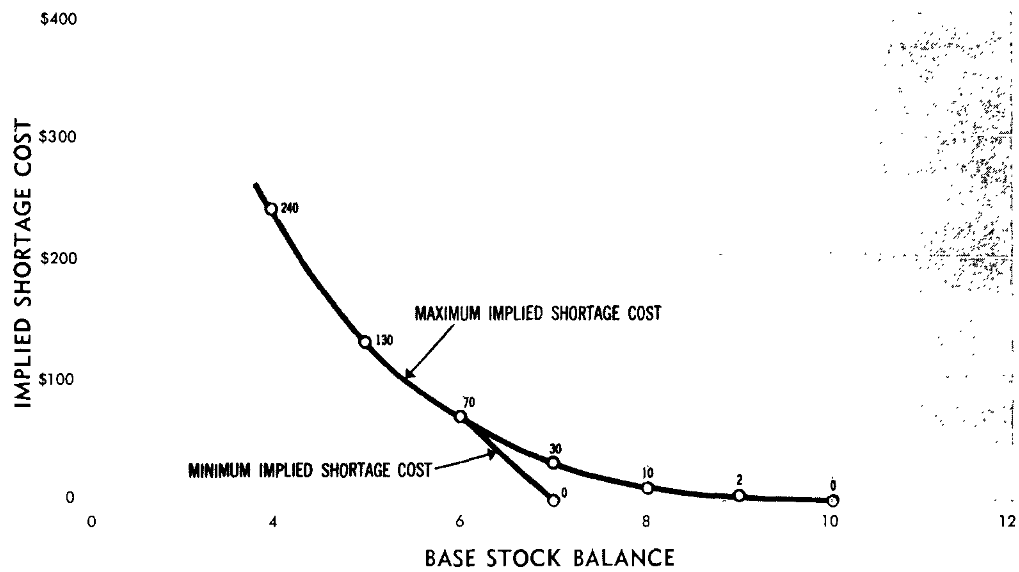

EXHIBIT 9. MAXIMUM AND MINIMUM IMPLIED SHORTAGE COST FOR DEPOT

two curves produced by adding in the two different implied shortage costs are shown in Exhibit 10. On each of the resultant curves a depot stock control level and a depot reorder point are determined. Thus, four levels emerge from the depot computation: the depot maximum stock control level and reorder point and the depot minimum stock control level and reorder point.

Neither the maximum nor minimum stock levels represents the least-cost inventory policy for the depot. The true optimum policy for the depot is not known; it may not even be a simple stock control level - reorder point relationship. It has been proved, however, that the expected cost of the unknown optimum policy is bracketed by the expected costs of the maximum and minimum depot policies provided the implied

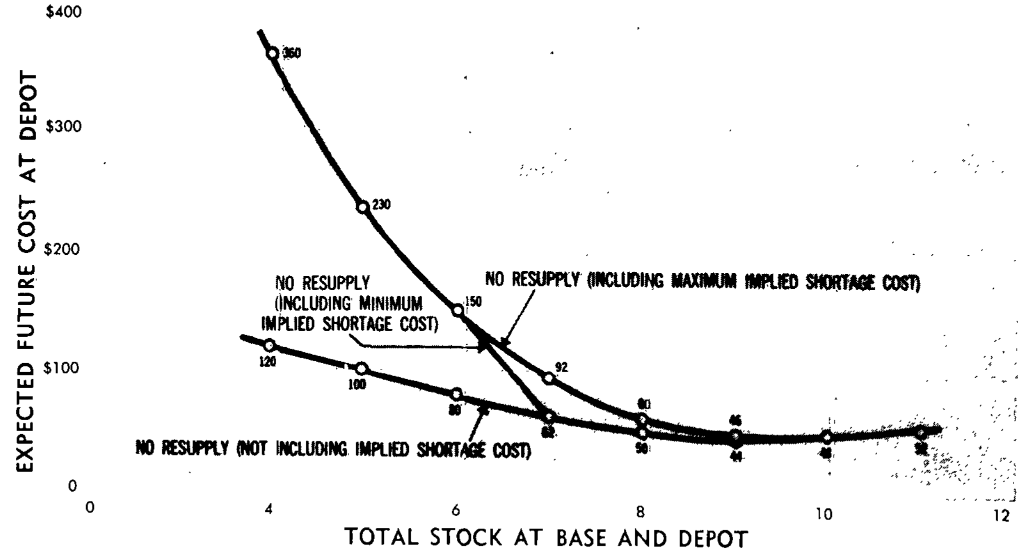

EXHIBIT 10. NO RESUPPLY CURVE FOR DEPOT (INCLUDING AND NOT INCLUDING MAXIMUM AND MINIMUM IMPLIED SHORTAGE COST)

shortage costs used in computing the two policies are correct.[1] The two implied shortage costs, of course, are never really correct since the base is not below its reorder point at all times or above it at all times.

But these two expected costs are useful even though they represent hypothetical situations. The expected cost, for instance, associated with the maximum depot policies represents the <u>maximum possible</u> cost expected from using the depot maximum policies. Similarly, the expected cost associated with the depot minimum policies represents a <u>minimum possible</u> cost expected from using the depot minimum policies. Since

[1]Scarf has proved this relationship and will publish it in a subsequent Planning Research Corporation report. The relationship holds only for a straight-line, vertical, multi-echelon supply system. While it has not been proved mathematically, it is felt that the relationship will hold for a multi-base case when trans-shipment between bases is infrequent (as it seems to be in practice).

the unknown optimum cost lies somewhere between the two possibilities it is feasible to use the depot maximum policies with the assurance that even though the actual expected costs will be greater than optimum, the margin by which they will be greater is at least no larger than the margin between the maximum and minimum possible expected costs.

Unfortunately, similar results do not hold if the depot minimum policies are used. It is not possible to use the depot minimum policies and be assured that the actual costs will lie in the interval between the maximum and minimum possible expected costs. This is because the depot minimum policies lead to lower inventory levels than the maximum policies and, in some situations, could therefore lead to excessive shortage costs. Graphic representations of the relationships of the various costs to one another are shown in Exhibit 11; definitions applying to the various costs are given in Exhibit 12.

The answer to the problem of whether to use the depot maximum or minimum policy is clear. For purposes of quantitative analysis, the depot maximum policy must be used because the margin by which the expected costs are greater than optimum may be calculated, whereas it cannot easily be determined if the depot minimum policy is used. Additionally, this calculated margin is the maximum possible margin and represents, therefore, an overstatement of the detriment caused by use of the depot maximum policies.

There will be some items in any inventory for which the depot maximum policy is clearly the wrong policy. These are the very inexpensive items for which it is economical to purchase a supply to last for several time periods. For these items the minimum policy is the correct one to use. However, in an effort to place an upper limit on the margin by which actual costs exceed optimum costs, it is assumed throughout the subsequent analysis that the depot maximum policy is always used.

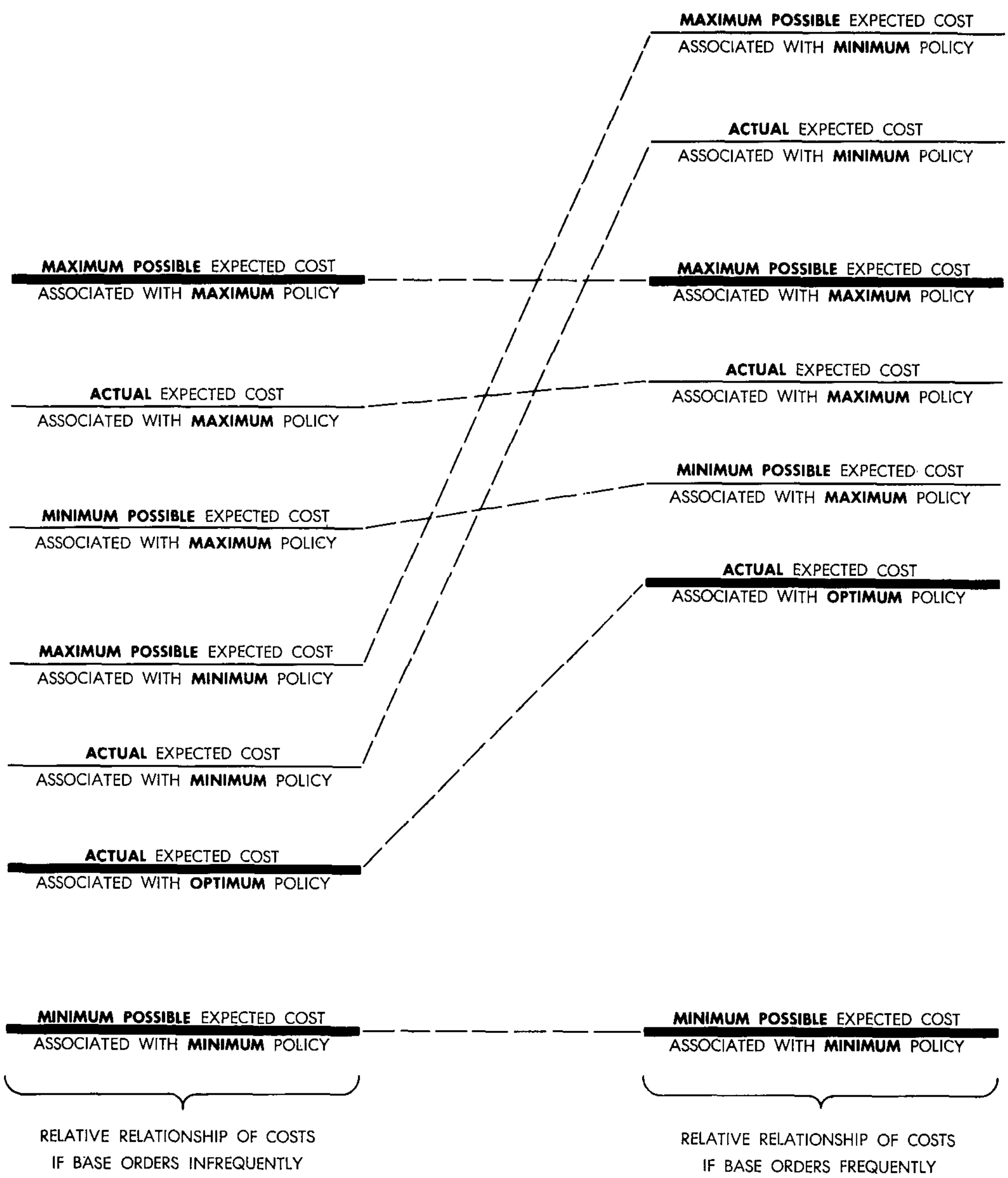

EXHIBIT 11. RELATIVE COST RELATIONSHIPS

EXHIBIT 12. COST DEFINITIONS

MAXIMUM POSSIBLE Expected Cost Associated with MAXIMUM Policy:

Total cost expected if the depot maximum policy is used and the base is at or below its reorder point at the start of every period.

MINIMUM POSSIBLE Expected Cost Associated with MAXIMUM Policy:

Total cost expected if the depot maximum policy is used and the base is never at or below its reorder point.

MAXIMUM POSSIBLE Expected Cost Associated with MINIMUM Policy:

Total cost expected if the depot minimum policy is used and the base is at or below its reorder point at the start of every period.

MINIMUM POSSIBLE Expected Cost Associated with MINIMUM Policy:

Total cost expected if the depot minimum policy is used and the base is never at or below its reorder point.

ACTUAL Expected Cost Associated with MAXIMUM Policy:

Total cost expected if the depot maximum policy is used and the base places an order whenever it is at or below its reorder point.

ACTUAL Expected Cost Associated with MINIMUM Policy:

Total cost expected if the depot minimum policy is used and the base places an order whenever it is at or below its reorder point.

ACTUAL Expected Cost Associated with OPTIMUM Policy:

Total cost expected if the unknown optimum policy were used at the depot and the base placed an order whenever it was at or below its reorder point.

III. SENSITIVITY OF POLICIES AND COSTS TO PARAMETER VALUES

The sensitivity of policies and costs to parameter values is investigated to identify regions in the parameter space where the maximum and minimum depot policies lead to expected costs that are close together; in these regions the maximum and minimum depot policies are good approximations to the unknown optimum policy. Conversely, it is desired to know which regions will yield poor approximations to the optimum policy.

The sensitivity study was executed by computing the policies and their expected costs on an IBM 7090 computer.[1]

A. Nominal Case

In order to investigate the effects of parameter values on policies and costs it is necessary to fix on a nominal case with nominal parameter values. The parameters are then systematically varied, one at a time, about their nominal values. In this way the effect of each parameter may be studied independently of the effects of all other parameters.

The nominal parameter values chosen for this case are shown in Exhibit 13. Note that the supply structure is of the simplest multi-echelon type: a single base that obtains all its resupply from a single depot. The depot is assumed to obtain its resupply from an infinite source, and the mean demand is constant from period to period.

B. Results of Sensitivity Study

The sensitivity of policies and costs to nine different parameters is investigated. The parameters are as follows:

1. Mean demand per period
2. Base fixed order cost
3. Depot fixed order cost
4. Base shortage cost

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[1]The computer program was written by J. R. Vander Veer.

5. Depot shortage cost
6. Base unit cost
7. Number of bases (total base demand constant)
8. Distribution of demand among bases (3 bases)
9. Depot leadtime

The results are presented in two graphs for each parameter. The first graph shows the steady-state stock levels for both the base and depot. The second graph shows the total expected costs (that is, expected unit, reorder, shortage, and holding costs) per time period for both the maximum and minimum depot policies. When the maximum policies are used it is assumed that the maximum implied shortage cost is applicable, and when the minimum policies are used it is assumed that the minimum implied shortage cost is applicable.

DEPOT

FIXED ORDER COST = $40
SHORTAGE COST = 0
UNIT COST = $10
HOLDING COST RATE = 2% OF UNIT COST PER PERIOD
INTEREST RATE = 10% PER PERIOD
PROCUREMENT LEADTIME = 1 PERIOD

BASE

MEAN DEMAND = 5 PER PERIOD (POISSON DISTRIBUTED)
FIXED ORDER COST = $20
SHORTAGE COST = $100 PER PERIOD
UNIT COST = $1
HOLDING COST RATE = 2% OF UNIT COST PER PERIOD
INTEREST RATE = 10% PER PERIOD
PROCUREMENT LEADTIME = 1 PERIOD

EXHIBIT 13. PARAMETER VALUES FOR NOMINAL CASE

Also shown on the second graph for each parameter is the percent difference between the costs associated with the maximum and minimum depot policies. The percent difference is computed with respect to the depot minimum cost as follows:

$$\begin{pmatrix}\text{percent}\\ \text{difference}\end{pmatrix} = \frac{(\text{max level cost}) - (\text{min level cost})}{(\text{min level cost})}$$

This percent difference has an alternate definition: it is the maximum percent difference between the costs associated with the depot maximum policy and those associated with the unknown optimum policy. In short, this alternate definition assumes the optimum cost to be identical with the cost associated with the depot minimum levels.

1. Mean Demand per Period

Exhibits 14 and 15 show that all levels and costs increase as the mean demand per period increases. Exhibit 15 also shows that the approximation to the unknown optimum policy degenerates as the mean demand approaches zero.

Note that for these particular parameters, the depot stock control level in Exhibit 14 is always less than the base stock control level. The base, therefore, will never be stocked up to its stock control level. This happens because the base stock control level is determined on the basis of an infinite supply of \$1.00 items that cost \$.02 per period to hold when the supply, in reality, is not infinite because it is stocked with \$10.00 items that cost \$.20 per period to hold. Units are so expensive for the depot to purchase and hold that it is willing to be charged base implied shortage costs rather than stock the number of units the base requires.

An important result of the relativity of the base and depot stock control levels is that the depot will rarely carry stock on hand. When stock is received at the depot it will be shipped immediately to the always-needy base.

While the amount that the base stock control level exceeds the depot stock control level is not predictable, the depot reorder points in Exhibit 14

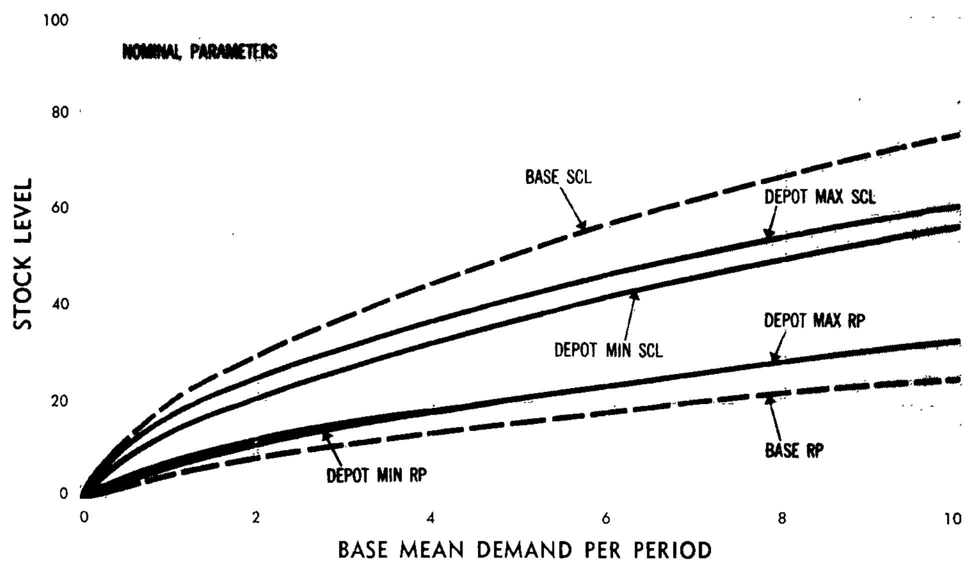


EXHIBIT 14. EFFECT OF BASE MEAN DEMAND PER PERIOD ON STOCK LEVELS

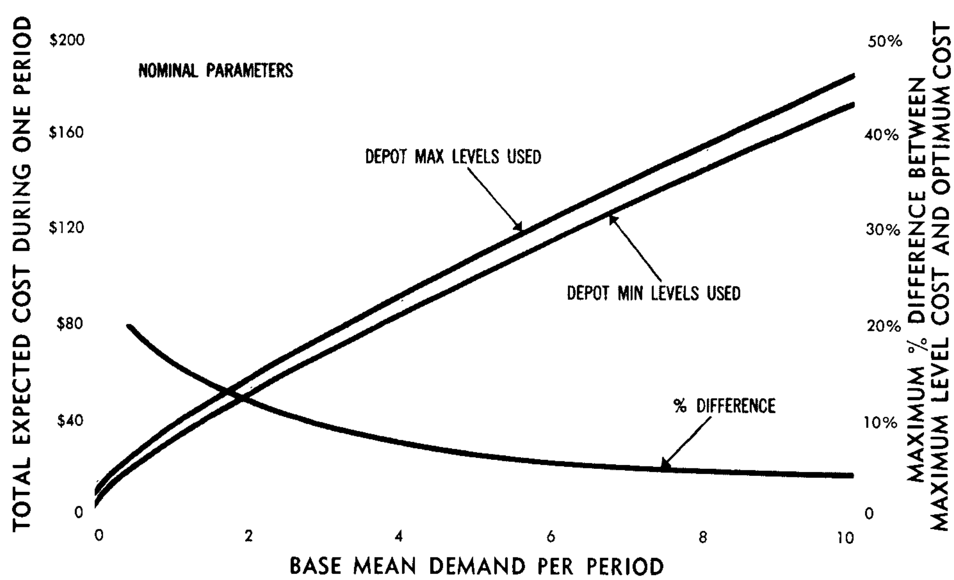


EXHIBIT 15. EFFECT OF MEAN DEMAND PER PERIOD ON COSTS

exceed the base reorder point by approximately the average amount demanded during the depot leadtime. This means that the depot usually will receive its order by the time the base places its order on the depot.

One other salient point about Exhibit 14 is that the depot maximum and minimum reorder points are nearly identical. This will usually be the case when the base shortage is significantly greater than the depot shortage cost (as it is in reality). The dominant base shortage cost causes the depot reorder points to be dependent mainly on the base implied shortage cost. The maximum and minimum implied shortage costs are identical at and below the base reorder point, and the values of the maximum implied shortage cost above the base reorder point are sufficiently low that little additional effect on the depot maximum reorder point is produced.

The three points outlined above--the restriction of base stock levels by the depot, the depot reorder points exceeding the base reorder point by the average amount demanded during the depot leadtime, and the identical depot maximum and minimum reorder points--appear repeatedly throughout the remaining portion of the sensitivity study.

2. Base Fixed Order Cost

Exhibit 16 shows that the base stock control level increases rapidly and the base reorder point remains nearly constant as the base fixed order cost increases. The increasing base stock control level widens the spread of possible total base and depot stock positions against which the maximum implied shortage cost is charged. This causes the depot maximum stock control level in Exhibit 16 to increase. The depot minimum stock control level remains unaffected, because the minimum implied shortage cost depends only on the unchanging base reorder point.

Exhibit 17 illustrates that the approximation is good when the base unit cost dominates the base fixed order cost, but degenerates as the base fixed order cost becomes large. The percent difference curve starts from the origin because the maximum and minimum implied shortage costs are identical when the base fixed order cost is zero.

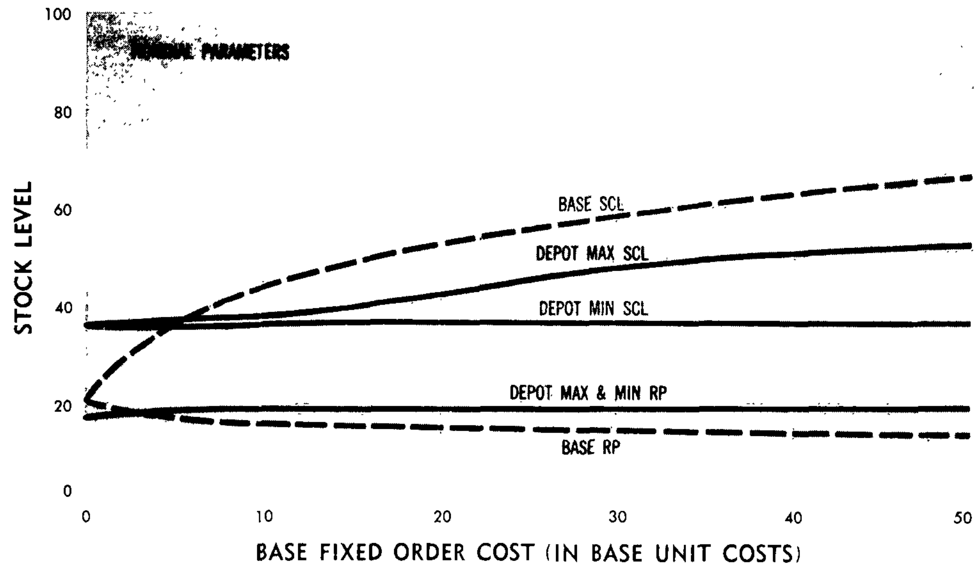


EXHIBIT 16. EFFECT OF BASE FIXED ORDER COST ON STOCK LEVELS

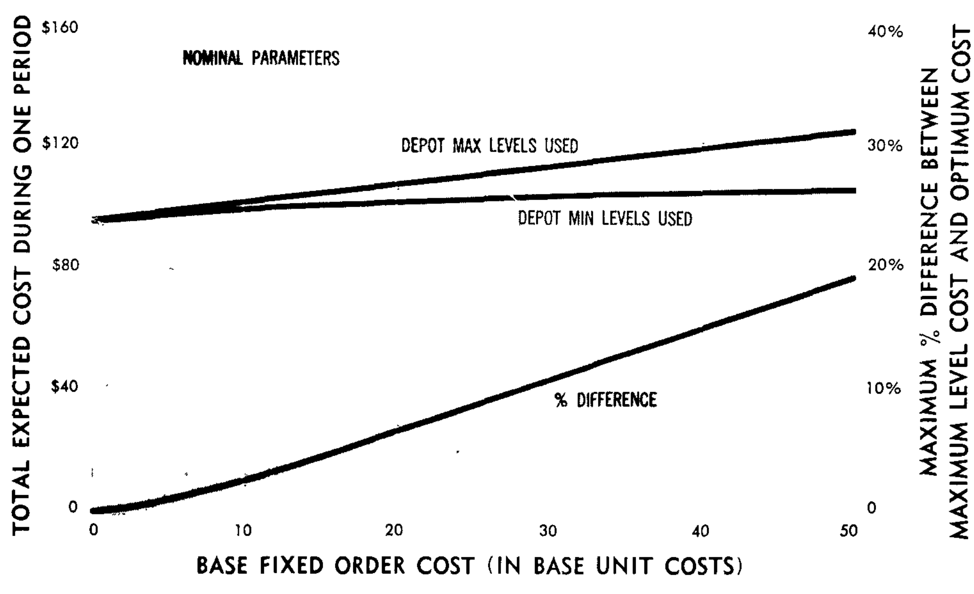


EXHIBIT 17. EFFECT OF BASE FIXED ORDER COST ON COSTS

3. Depot Fixed Order Cost

The pattern formed by the depot stock levels in Exhibit 18 is very similar to the one formed by the base stock levels in Exhibit 16. This is because the base stock levels and implied shortage costs are unaffected by changes in the depot fixed order cost, and the depot views the implied shortage cost as a constant shortage cost much the same as the base views its $100 shortage cost. The resulting stock levels, therefore, react to changes in the depot fixed order cost in a manner similar to the base stock level reaction to a changing base fixed order cost.

The effect of the depot fixed order cost on costs is more marked than that of the base fixed order cost, but the effect on the percent difference between the maximum and minimum costs is just the opposite. Exhibit 19 shows that the approximation to the least-cost policy becomes better as the depot fixed order cost increases. This is because a high

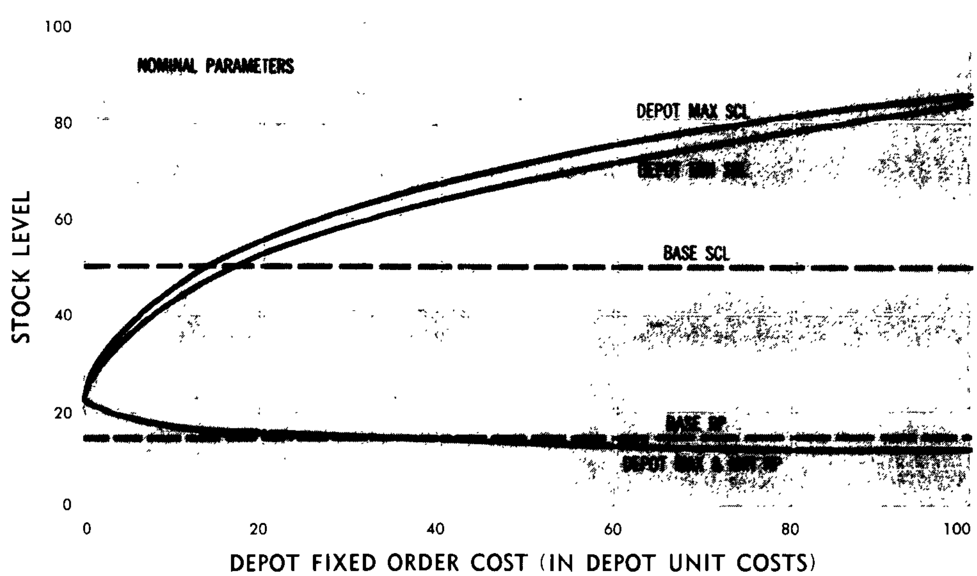


EXHIBIT 18. EFFECT OF DEPOT FIXED ORDER COST ON STOCK LEVELS

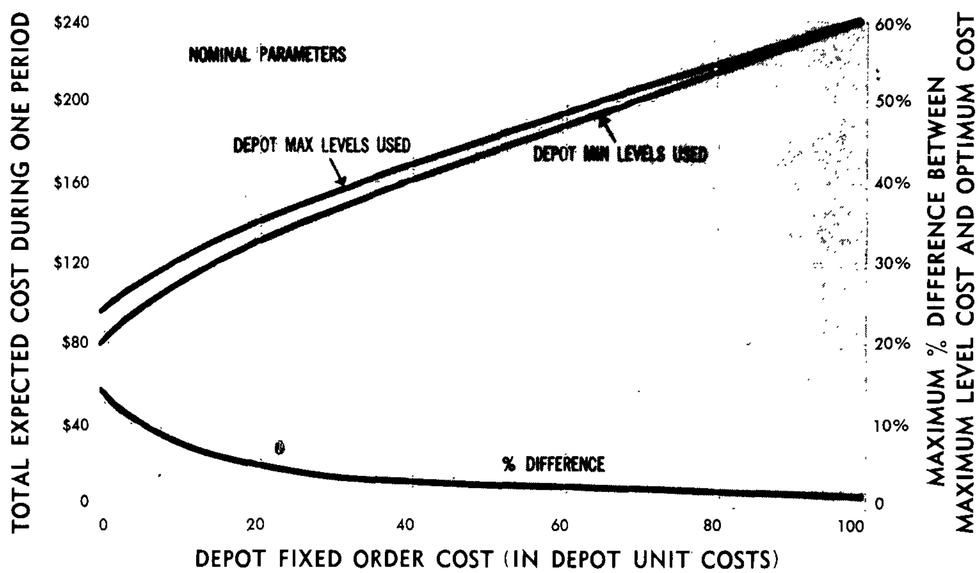


EXHIBIT 19. EFFECT OF DEPOT FIXED ORDER COST ON COSTS

depot fixed order cost causes the depot to carry more inventory, thereby reducing the chance of base orders not being filled and mitigating the influence of the base implied shortage cost.

4. Base Shortage Cost

As the base shortage cost increases, all levels rise from zero to some reasonably constant value as shown in Exhibit 20. The constant value is approached because of the decreasing marginal shortage protection offered by additional units of stock.

The base shortage cost has little effect on the quality of the approximation to the optimum policy, except when the shortage cost approaches the unit cost as shown in Exhibit 21.

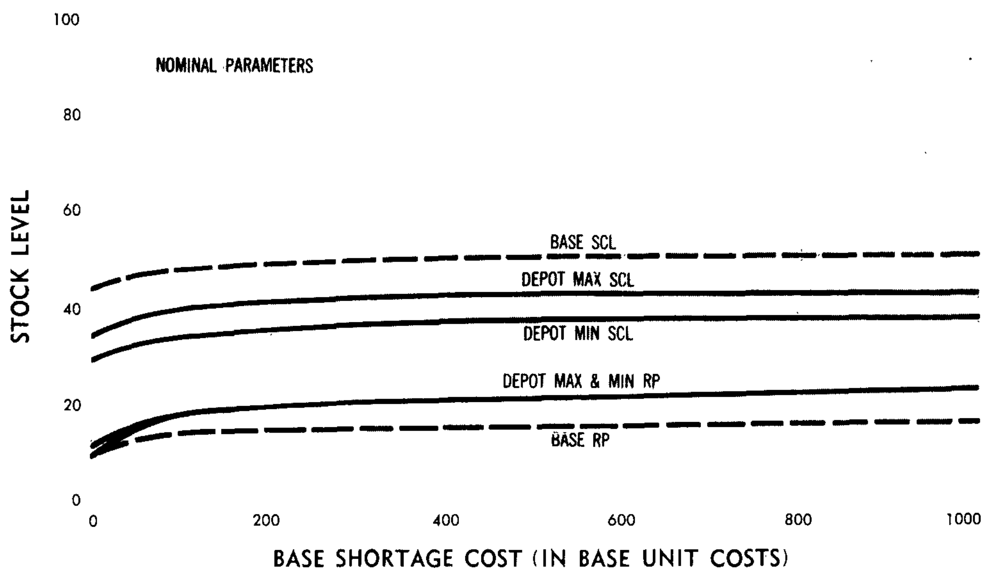


EXHIBIT 20. EFFECT OF BASE SHORTAGE COST ON STOCK LEVELS

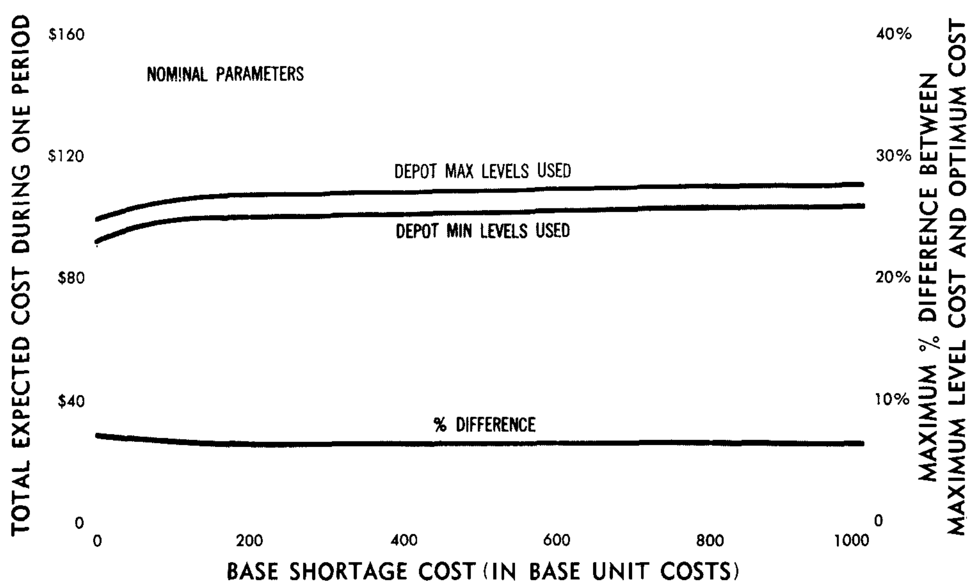


EXHIBIT 21. EFFECT OF BASE SHORTAGE COST ON COSTS

5. Depot Shortage Cost

For this particular region in the parameter space, the depot shortage cost has absolutely no effect on either the policies or the costs as illustrated in Exhibits 22 and 23. This is because the ratio of shortage cost to unit cost (100) at the base is high enough to cause the depot, through the implied shortage cost, to purchase units to protect the base from the shortages even when the depot shortage cost is zero. Therefore, additional depot shortage costs, because of the decreasing marginal protection from shortages offered by additional units of stock, do not cause increased stockage.

If the ratio of base shortage cost to unit cost were lower, a changing depot shortage cost would certainly have an effect on the policies and costs when it approached a sufficiently large value. Such a situation, however, is unrealistic and would seldom be encountered in actual practice.

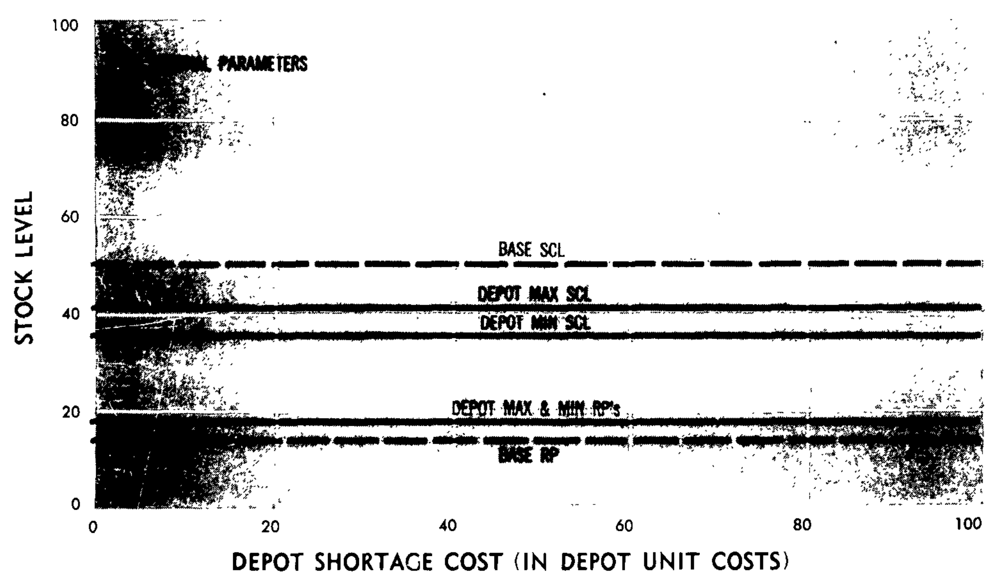


EXHIBIT 22. EFFECT OF DEPOT SHORTAGE COST ON STOCK LEVELS

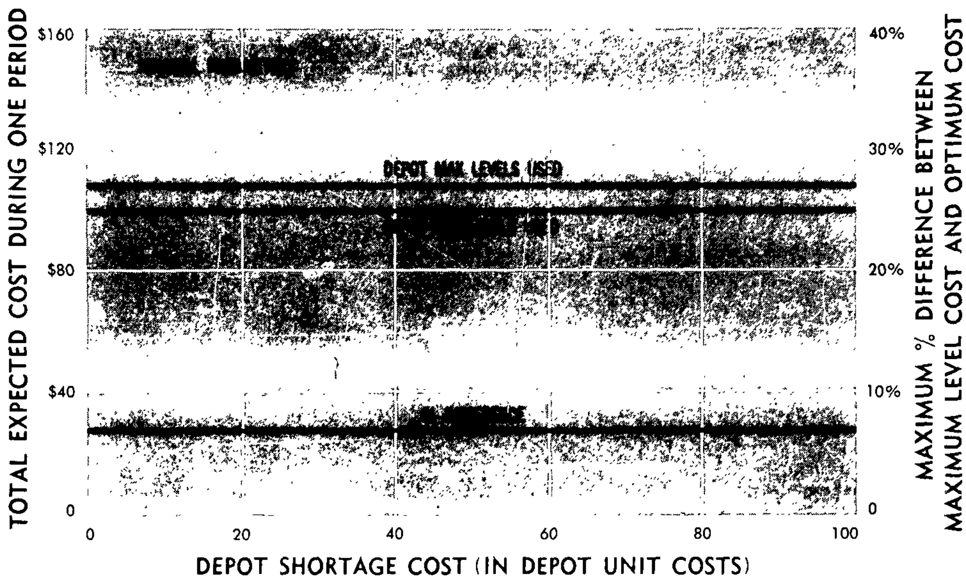


EXHIBIT 23. EFFECT OF DEPOT SHORTAGE COST ON COSTS

6. Base Unit Cost

Since the depot purchases items from a vendor, the base unit cost is really only the cost of transporting the item from depot to base. When the transportation cost is a small fraction of the depot's cost, as is usually the case, the base stock level will be restricted by the depot as shown in Exhibit 24. This is because the base stock control level is rising rapidly while the base reorder point is rising only slightly. The number of stock positions greater than the base reorder point for which maximum implied shortage costs are charged is also increasing, but the values of these maximum implied shortage costs are decreasing because of the slightly increasing base reorder point. The net effect is that the depot maximum stock control level increases, but at a slower rate than the base

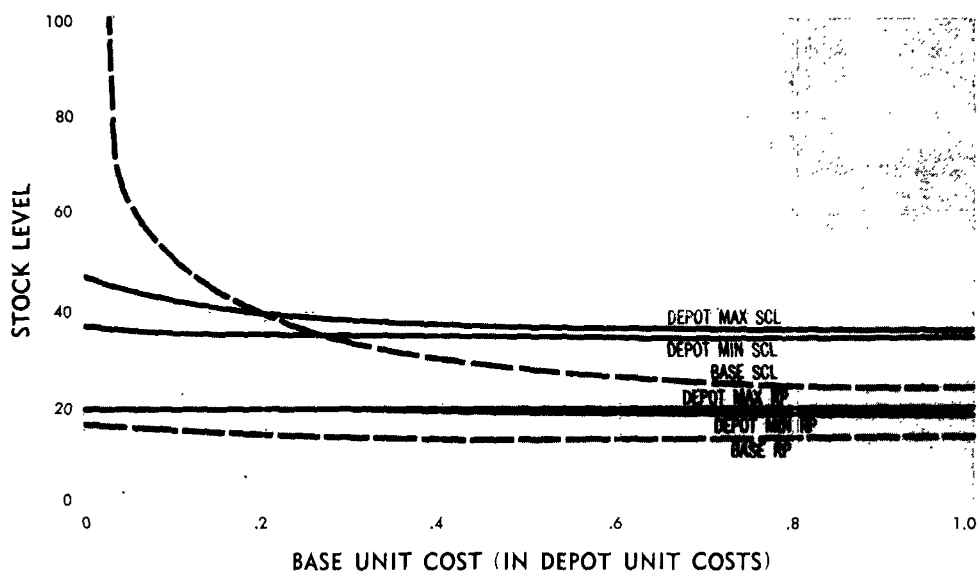


EXHIBIT 24. EFFECT OF BASE UNIT COST ON STOCK LEVELS

stock control level; the depot minimum stock control level, because it is independent of the rapidly increasing base stock control level, increases at a rate even slower than the depot maximum stock control level.

Exhibit 25 shows that the costs increase as the base unit cost increases. This is because the total cost of the 5 units demanded on the average during a period is increasing. Also, because reorder points and stock control levels are decreasing, increased shortage and fixed order costs are starting to dominate the holding cost saved. Exhibit 25 also illustrates that the approximation to the optimum policy becomes poor as the transportation cost decreases with respect to the depot unit cost.

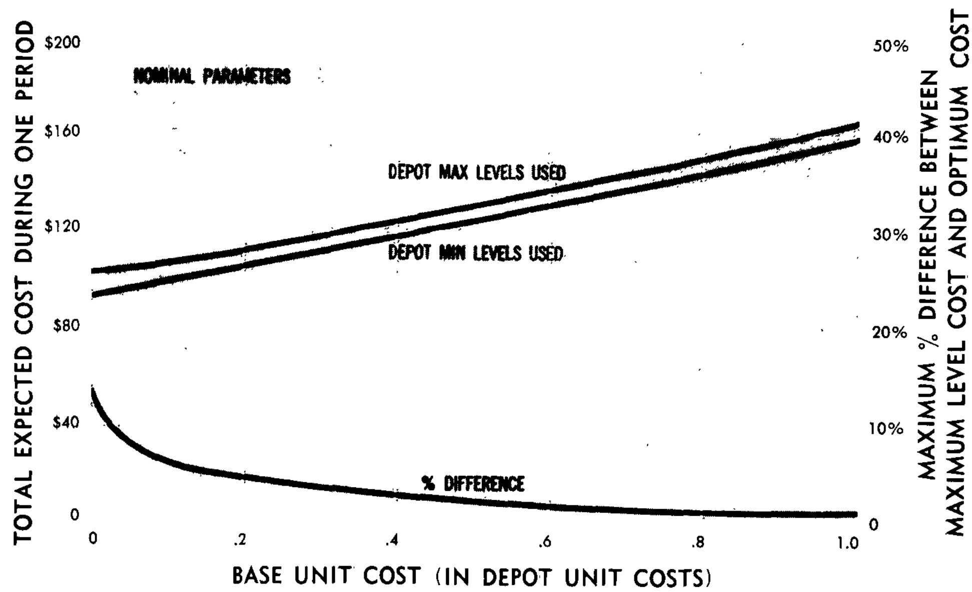


EXHIBIT 25. EFFECT OF BASE UNIT COST ON COSTS

7. Number of Bases

Exhibit 26 shows that the depot maximum levels follow the same trend as the sum of the base stock control levels, and the depot minimum levels follow the same trend as the sum of the base reorder points. In both cases, as the number of bases increases, the depot minimum order quantity (stock control level less reorder point) remains fairly constant; this is because the depot is experiencing the same average total base demand of 5 per period irrespective of the number of bases.

The depot maximum and minimum reorder points in Exhibit 26, unlike those in the previous graphs, are separating as the parameter of interest increases. The depot minimum reorder point follows the trend of the sum of the base reorder points because the implied shortage cost, on which it is dependent, is solely dependent on the sum of the base reorder

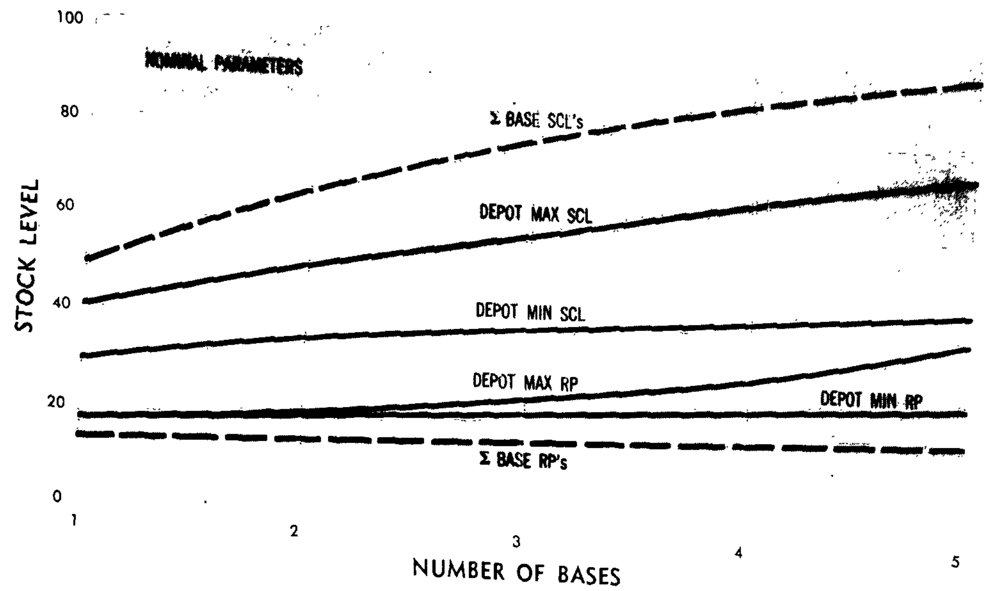


EXHIBIT 26. EFFECT OF NUMBER OF BASES ON STOCK LEVELS (TOTAL BASE DEMAND CONSTANT)

points. The depot maximum reorder point is *increasing* at a faster rate because the maximum implied shortage cost is becoming chargeable to more stock positions above the sum of the base reorder points, and the values of the implied shortage costs in this region are increasing because the shortage risk taken by each base increases when the mean demand decreases.

Exhibit 27 shows that the approximation to the unknown optimum policy degenerates rapidly as the number of bases increases.

8. <u>Distribution of Demand Among Bases</u>

Exhibit 28 shows that for other than a greatly imbalanced distribution of demand among bases, the stock levels are fairly insensitive to such distribution. The reason that the stock levels decrease at the left

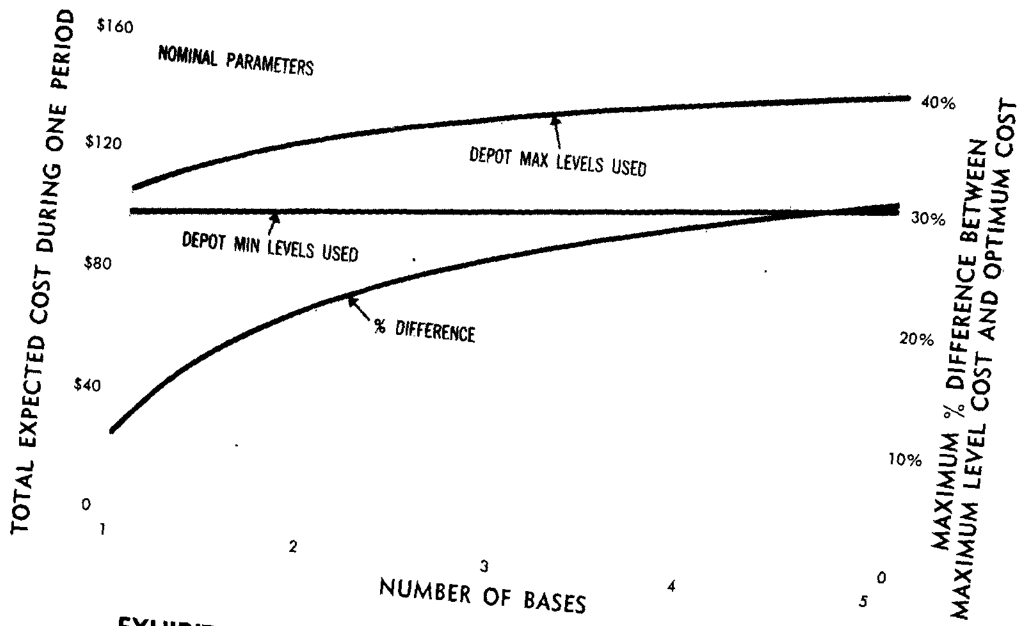


EXHIBIT 27. EFFECT OF NUMBER OF BASES ON COSTS (TOTAL BASE DEMAND CONSTANT)

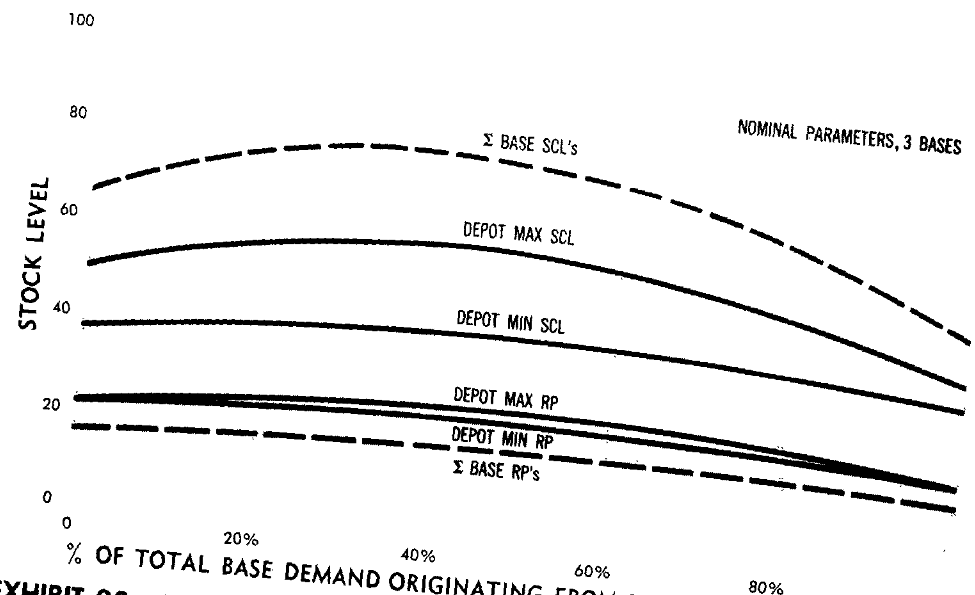


EXHIBIT 28. EFFECT OF DISTRIBUTION OF DEMAND AMONG BASES ON STOCK LEVELS (TOTAL BASE DEMAND CONSTANT)

*REMAINING DEMAND DIVIDED EVENLY BETWEEN OTHER TWO BASES.

and right extremities of the graph is that the demand configuration seems fewer than three bases: two bases at the left extremity and one at the right extremity. As shown in Exhibit 26, fewer bases lead to lower stock levels.

Exhibit 29 shows that when the imbalance of demand is greatest, the approximation to the optimum policy improves. This also is due to the demand seeming to originate from fewer bases; the same effect is shown in Exhibit 27.

9. Depot Leadtime

The base stock levels and implied shortage costs, of course, are independent of variations in the depot leadtime as shown in Exhibit 30. The depot stock levels, therefore, appear as they would in the usual single-echelon case; that is, the depot reorder point is very sensitive to leadtime while the minimum order quantity is not.

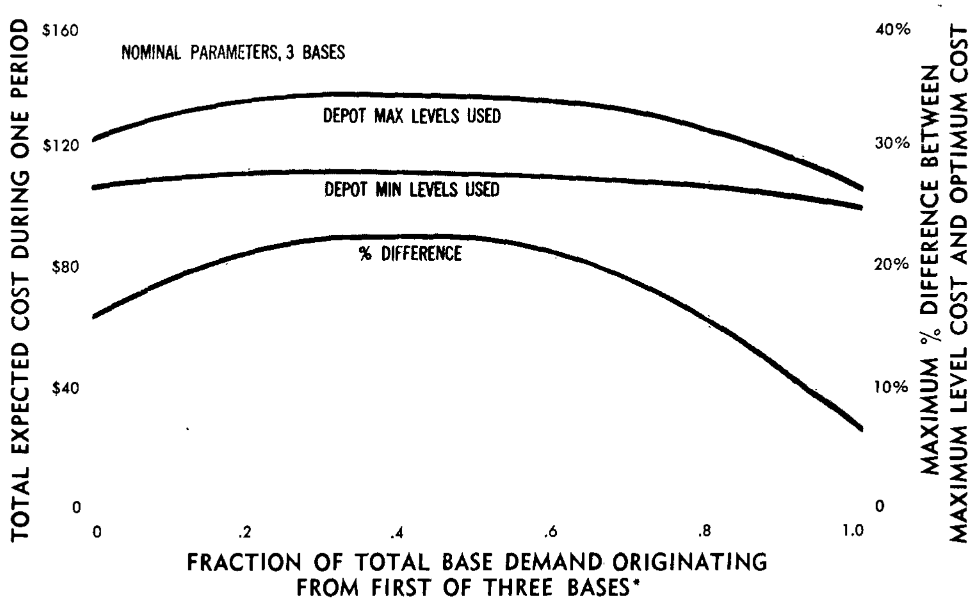


EXHIBIT 29. EFFECT OF DISTRIBUTION OF DEMAND AMONG BASES ON COSTS (TOTAL BASE DEMAND CONSTANT)

*REMAINING DEMAND DIVIDED EVENLY BETWEEN OTHER TWO BASES.

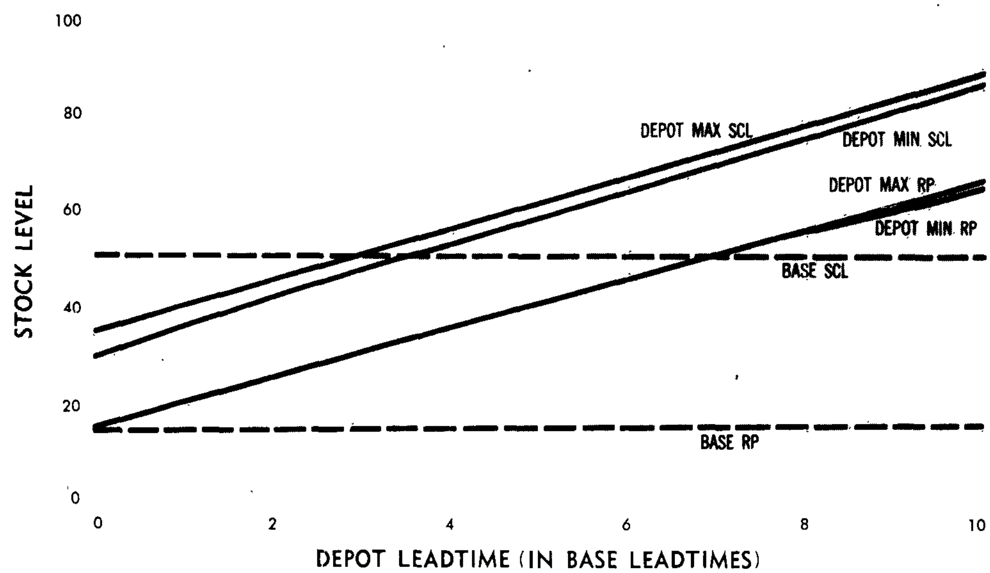


EXHIBIT 30. EFFECT OF DEPOT LEADTIME ON STOCK LEVELS

The approximation to the unknown optimum policy is good for all ranges of the depot leadtime, as illustrated in Exhibit 31, and improves as the depot leadtime increases.

10. Summarized Results

The general results pertinent to the quality of the approximation to the optimum policy are shown in Exhibit 32. This information is obtained from the curves of average percent difference between the expected costs of using the maximum and minimum depot policies. A high average percent difference means that the approximation to the least-cost policy is poor, and vice versa. Note that neither the base nor depot shortage cost appears in this listing; this is because changes in these parameters had no appreciable effect on the average percent difference.

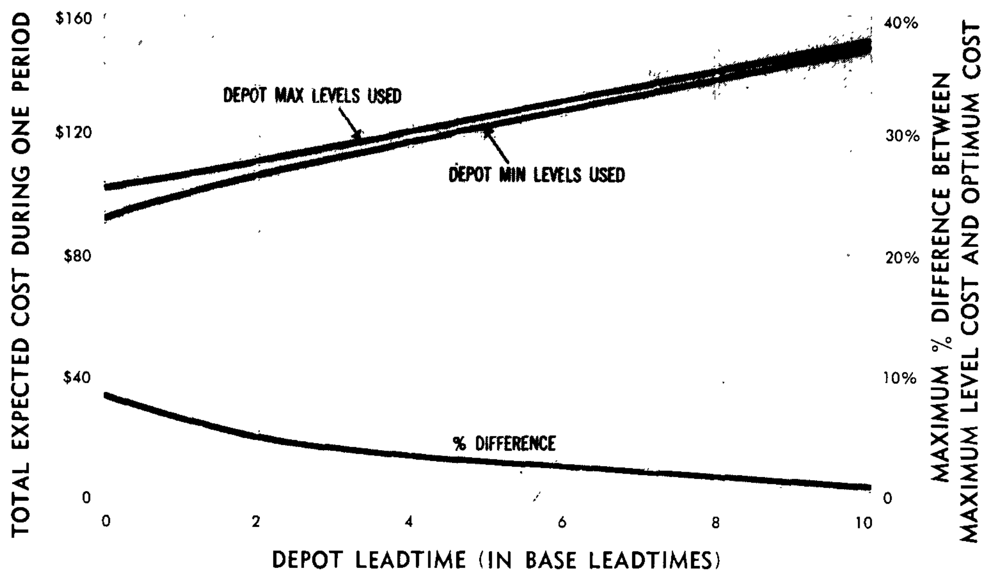


EXHIBIT 31. EFFECT OF DEPOT LEADTIME ON COSTS

EXHIBIT 32. SUMMARY OF PARAMETER VALUES LEADING TO GOOD AND POOR APPROXIMATIONS TO THE OPTIMUM POLICY

Parameter Values Leading to Good Approximations	Parameter Values Leading to Poor Approximations
High Mean Demand Per Period	Low Mean Demand Per Period
Low Base Fixed Order Cost	High Base Fixed Order Cost
High Depot Fixed Order Cost	Low Depot Fixed Order Cost
High Base Unit Cost	Low Base Unit Cost
Few Bases	Many Bases
Imbalance of Demand Among Bases	Balance of Demand Among Bases
Long Depot Leadtime	Short Depot Leadtime

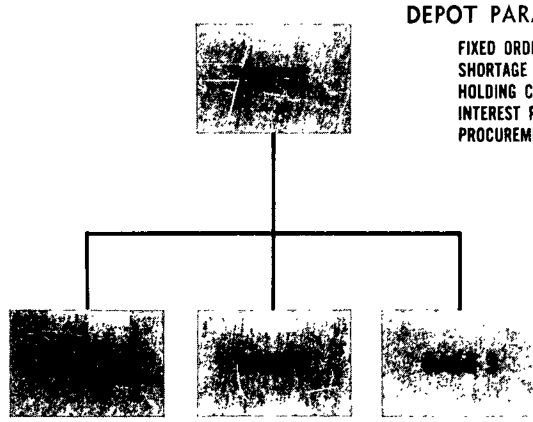
IV. APPLICATION OF POLICIES

The practicality of application of the approximate policies to actual inventories rests on a demonstration that such application would result in a cost reduction. The proper way to evaluate the potential cost savings attributable to the approximate policies would be to collect data in the field. Such data would include the characteristics of the particular multi-echelon supply system being studied, the characteristics of the items in inventory, and the policies presently used to manage the items. Analysis of such data would yield the present inventory costs generated in the system as well as the costs that would be generated if the approximate policies were implemented.

A different tack is taken in this evaluation, since an extensive data collection program is not warranted at this time. A hypothetical multi-echelon supply system is constructed and stocked with a realistic inventory of items. The costs of managing the inventory with the depot maximum and minimum levels are computed and compared. In this way, assuming that the depot maximum levels are used for the entire inventory, the cost increase over the <u>optimum</u> cost is estimated (assuming that optimum cost and depot minimum level cost are equal). Then, because the cost increase over optimum attributable to <u>traditional</u> inventory policies is roughly known through experience, the potential cost savings attributable to the approximate policies can be estimated.

The hypothetical multi-echelon supply system and pertinent parameter values chosen for this evaluation are shown in Exhibit 33. The parameter values are chosen to reflect values that might be encountered in the Navy supply system.

Mean demand and unit cost are notably missing from Exhibit 33. Data from an actual Navy inventory was used to set the values of these parameters. The inventory consists of 8,813 items of N cognizance material (electronics items) that were stocked at the Naval Shipyard, Mare Island, California, in the first quarter of the calendar year 1960. A joint distribution of unit cost and mean demand per quarter for the 8,813 items



DEPOT PARAMETERS

FIXED ORDER COST = $20
SHORTAGE COST = $50
HOLDING COST RATE = 0.5% PER MONTH
INTEREST RATE = 2% PER MONTH
PROCUREMENT LEADTIME = 6 MONTHS

BASE PARAMETERS

FIXED ORDER COST = $5
SHORTAGE COST = 10 TIMES THE BASE UNIT COST
HOLDING COST RATE = 0.5% PER MONTH
INTEREST RATE = 2% PER MONTH
PROCUREMENT LEADTIME = 1 MONTH

EXHIBIT 33. MULTI-ECHELON SUPPLY SYSTEM AND PARAMETER VALUES USED IN FEASIBILITY EVALUATION

is shown in Exhibit 34. The purpose of using this data in the feasibility evaluation is to portray a realistic inventory composition, and <u>not</u> to evaluate implementation of the policies at NSY Mare Island.

Certain assumptions have to be made concerning unit cost and mean demand. With respect to the unit cost, it is assumed that it was divided 80 percent and 20 percent, 80 percent being the purchase cost paid by the depot and 20 percent being the transportation cost to a base. With respect to the mean demand, it is assumed to be divided evenly among the three bases and to be Poisson distributed during a quarter.

After the quarterly demands were converted to monthly demands, it was assumed that the average item in each cell exhibited parameters valued at the midpoint of the cell. The base policies and costs and the depot maximum and minimum policies and costs were then computed for as many cells

MEAN DEMAND PER QUARTER

UNIT COST	LESS THAN 1.0	1.0 - 10	10 - 100	100 - 1000	MORE THAN 1000	TOTALS
LESS THAN $0.10	383	118	69	29	10	589
$0.10 - $1.00	1,823	206	69	12	NO ITEMS IN CELL	2,110
$1.00 - $10	3,774	217	78	9	NO ITEMS IN CELL	4,078
$10 - $100	1,499	162	16	NO ITEMS IN CELL	NO ITEMS IN CELL	1,677
MORE THAN $100	339	19	1	NO ITEMS IN CELL	NO ITEMS IN CELL	359
TOTALS	7,798	722	233	50	10	8,813

EXHIBIT 34. JOINT DISTRIBUTION OF UNIT COST AND MEAN DEMAND FOR 8,813 N Cog ITEMS AT NSY MARE ISLAND

as possible. Because of computing limitations, policies and costs for several of the cells were not obtained. Fortunately, the elimination of these particular cells from the evaluation is not detrimental; because of the high- and low-value phenomena in the inventory, these cells account for only a small fraction of the total inventory cost.

Since the computed costs applied to the average item in the cell, it was possible to multiply these costs by the number of items in the cell to obtain the total inventory cost for the cell. The results of this operation are shown in Exhibits 35 and 36 for the maximum and minimum depot policies, respectively. The cell in the lower right-hand corner in each case gives the total inventory cost: $197,101 for the maximum policies and $192,347 for the minimum policies. The cost associated with the maximum policies is 2.5 percent greater than that associated with the

UNIT COST	LESS THAN 1.0	1.0 - 10	10 - 100	100 - 1000	MORE THAN 1000	TOTALS
LESS THAN \$0.10	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE	
\$0.10 - \$1.00	COMPUTATION NOT POSSIBLE	\$762	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE	NO ITEMS IN CELL	\$762
\$1.00 - \$10	\$13,284	\$3,943	\$9,661	COMPUTATION NOT POSSIBLE	NO ITEMS IN CELL	\$26,888
\$10 - \$100	\$32,933	\$21,562	\$17,756	NO ITEMS IN CELL	NO ITEMS IN CELL	\$72,251
MORE THAN \$100	\$62,990	\$23,362	\$10,848	NO ITEMS IN CELL	NO ITEMS IN CELL	\$97,200
TOTALS	\$109,207	\$49,629	\$38,265			\$197,101

(Column heading: MEAN DEMAND PER QUARTER)

EXHIBIT 35. TOTAL EXPECTED COST DURING ONE PERIOD (DEPOT MAXIMUM POLICIES USED AND MAXIMUM IMPLIED SHORTAGE COST APPLICABLE)

UNIT COST	LESS THAN 1.0	1.0 - 10	10 - 100	100 - 1000	MORE THAN 1000	TOTALS
LESS THAN \$0.10	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE	
\$0.10 - \$1.00	COMPUTATION NOT POSSIBLE	\$604	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE	NO ITEMS IN CELL	\$604
\$1.00 - \$10	\$12,680	\$3,435	\$9,308	COMPUTATION NOT POSSIBLE	NO ITEMS IN CELL	\$25,423
\$10 - \$100	\$30,325	\$21,089	\$17,707	NO ITEMS IN CELL	NO ITEMS IN CELL	\$69,121
MORE THAN \$100	\$62,990	\$23,362	\$10,847	NO ITEMS IN CELL	NO ITEMS IN CELL	\$97,199
TOTALS	\$105,995	\$48,490	\$37,862			\$192,347

(Column heading: MEAN DEMAND PER QUARTER)

EXHIBIT 36. TOTAL EXPECTED COST DURING ONE PERIOD (DEPOT MINIMUM POLICIES USED AND MINIMUM IMPLIED SHORTAGE COST APPLICABLE)

minimum policies. Therefore, use of the maximum policies would result in a maximum percent difference over the optimum cost of 2.5 percent.

Of course, the 2.5 percent would increase slightly if computation were possible for all the cells. The increase would probably be of the order of 0.5 percent, though, because of the relatively small costs associated with uncomputed cells. There are other factors, however, which will tend to decrease the 2.5 percent. For one thing, the minimum depot policy clearly would be better than the maximum policy for the inexpensive items. Also, the maximum implied shortage would really be applicable only for the most expensive items. These mitigating factors might reduce the 2.5 percent by 0.5 percent, thereby cancelling the effect of the uncomputed cells. The 2.5 percent increase over optimum cost, therefore, is a reasonable figure for this inventory.

The factor that keeps this percent difference so low is the high-value phenomenon in the inventory. This is illustrated in Exhibit 37, in which the percent difference in cost between the maximum and minimum policies is shown for each individual cell. By comparing these percent differences with the fraction of the inventory cost in corresponding cells in Exhibit 35, it is seen that the majority of the inventory cost carries small percent differences. In particular, the four cells with zero percent difference account for 58 percent of the total inventory cost.

Experience has shown that the percent increase over optimum cost resulting from the use of traditional policies is more than 2.5 percent. It probably averages 20 percent, and is as high as 30 percent in some cases. It is felt, then, that the potential saving which might be derived from implementation of the approximate policies is significant.

MEAN DEMAND PER QUARTER

UNIT COST	LESS THAN 1.0	1.0 - 10	10 - 100	100 - 1000	MORE THAN 1000
LESS THAN $0.10	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE
$0.10 - $1.00	COMPUTATION NOT POSSIBLE	28.2%	COMPUTATION NOT POSSIBLE	COMPUTATION NOT POSSIBLE	NO ITEMS IN CELL
$1.00 - $10	4.0%	14.0%	3.0%	COMPUTATION NOT POSSIBLE	NO ITEMS IN CELL
$10 - $100	0.0%	2.2%	0%	NO ITEMS IN CELL	NO ITEMS IN CELL
MORE THAN $100	0%	0%	0%	NO ITEMS IN CELL	NO ITEMS IN CELL

EXHIBIT 37. PERCENT DIFFERENCE BETWEEN COSTS ASSOCIATED WITH DEPOT MAXIMUM AND MINIMUM POLICIES FOR EACH CELL

V. AREAS FOR FURTHER RESEARCH

The following are a few suggested areas for further research; they do not appear in order of importance.

1. Explore means of implementation of the approximate policies; for example, tabled policies versus real-time calculated policies in a computer, centralized versus decentralized information, etc.

2. Analyze transition effects from present policies to approximate policies.

3. Define explicit regions in the parameter space where depot maximum policies are preferable to depot minimum policies and vice versa.

4. Find an "approximation to the approximate policies" that allows direct computation of policies in nondynamic cases, thereby avoiding the iterative dynamic programming procedure.

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2. Clark, A. J., and Scarf, H., "Optimal Policies for a Multi-Echelon Inventory Problem," PRC R-113, Los Angeles, California, Planning Research Corporation (July 1959).

3. Gradwohl, A. J., "Case Studies on the Multi-Echelon Inventory Problem," PRC R-133, Los Angeles, California, Planning Research Corporation (December 1959).